Problema 1864. Let ABC be a scalene triangle, I its incenter, and X, Y and Z the tangency points of its incircle $\mathcal{C}$ with sides BC, CA, and AB, respectively. Denote by $X' \neq A$, $Y' \neq B$, and $Z' \neq C$ the intersections of $\mathcal{C}$ with the circumcircles of triangles AIX, BIY, and CIZ, respectively. Prove that th lines AX', BY', and CZ' are concurrent.

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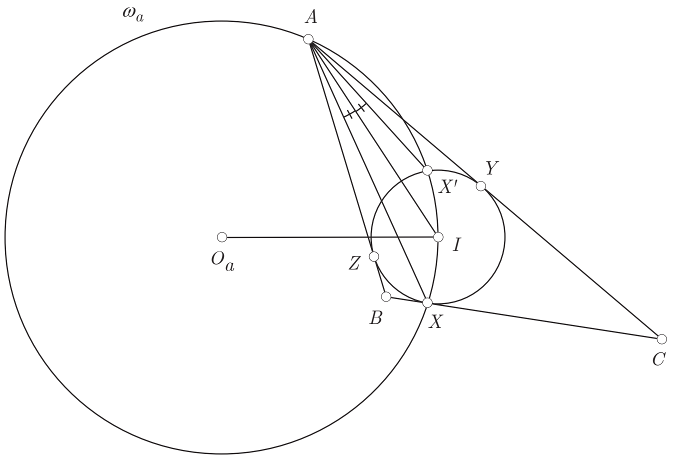
Solution by Ercole Suppa, Teramo, Italy

Denote by ω_a the circumcircle of triangle AIX and let O_a be its center.



Since $O_aI \perp XX'$, O_aI is the altitude through O_a of the isosceles triangle XO_aX'. Therefore O_aI is the angle bisector of $\angle XO_aX'$, so the arcs IX, IX' are congruent. Thus $\angle XAI = \angle X'AI$, i.e. the cevians AX and AX' of triangle ABC are isogonal conjugates and similarly it is shown that they are also BY, BY' and CZ, CZ'.

It is well known that AX, BY, CZ concur in a point J (called *Gergonne point* of $\triangle ABC$) and this implies that AX', BY', CZ' also concur in the point J' isogonal conjugate of J (see R. Honsberger, Episodes in Nineteenth and Twentieth Century Euclidean Geometry, Theorem 2, pag. 55). $\square$